Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Mathematics

Advanced
Paper 1: Pure Mathematics 1

Wednesday 6 June 2018 – Morning
Time: 2 hours

Paper Reference **9MA0/01**

1. Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1 - \cos 4\theta}{2\theta \sin 3\theta}$$

(3)

2. A curve C has equation

$$y = x^2 - 2x - 24\sqrt{x}, \qquad x > 0$$

(a) Find (i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 4$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

3.

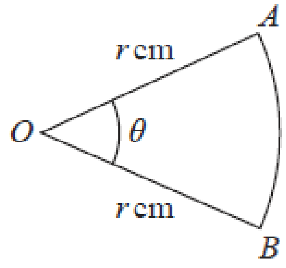


Figure 1

Figure 1 shows a sector AOB of a circle with centre O and radius r cm.

The angle AOB is θ radians.
The area of the sector AOB is 11 cm^2

Given that the perimeter of the sector is 4 times the length of the arc AB, find the exact value of r.

(4)

4. The curve with equation $y = 2\ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$

(2)

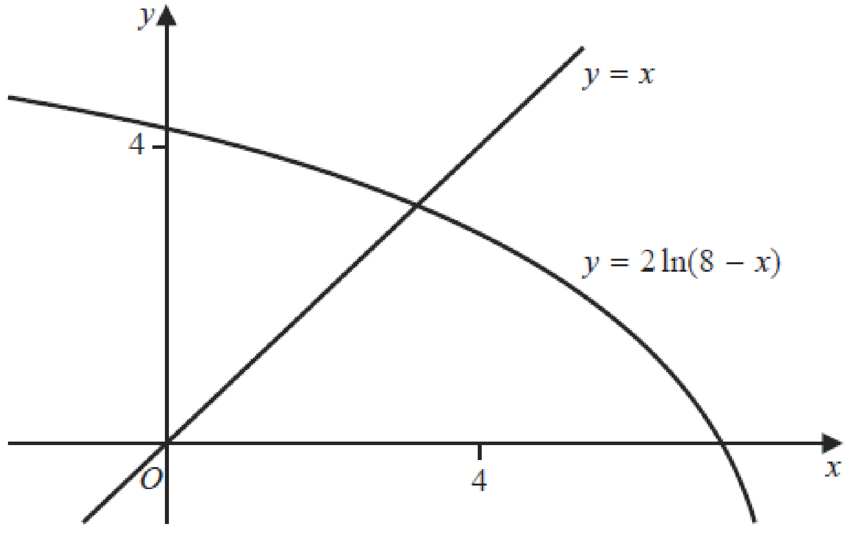


Figure 2

Figure 2 shows the graph of $y = 2\ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2\ln(8 - x_n), \quad n \in \mathbb{N}$$

in an attempt to find an approximation for α.

Using the graph and starting with $x_1 = 4$

(b) determine whether or not this iteration formula can be used to find an approximation for α, justifying your answer.

(2)

5. Given that

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}\theta} = \frac{A}{1 + \sin 2\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where A is a rational constant to be found.

(5)

6.

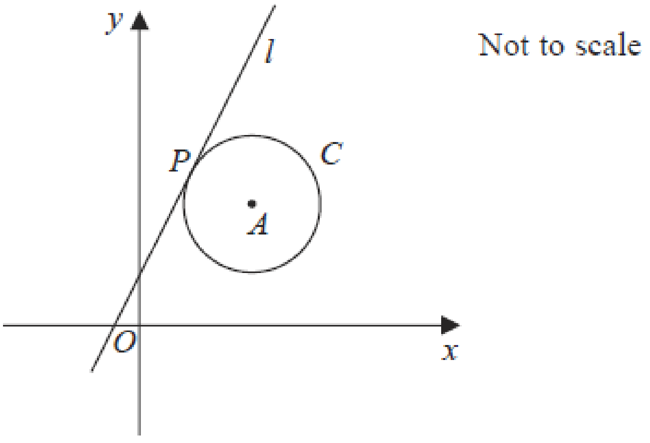


Figure 3

The circle C has centre A with coordinates $(7, 5)$.

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P, as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$

(3)

(b) Find an equation for C.

(4)

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to C.

(c) Find the value of the constant k.

(3)

7. Given that $k \in \mathbb{Z}^+$

(a) show that $\displaystyle\int_k^{3k} \frac{2}{(3x-k)}\,dx$ is independent of k,

(4)

(b) show that $\displaystyle\int_k^{2k} \frac{2}{(2x-k)^2}\,dx$ is inversely proportional to k.

(3)

8. The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)° \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.
The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.

(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.
(Solutions based entirely on graphical or numerical methods are not acceptable.)

(4)

9.

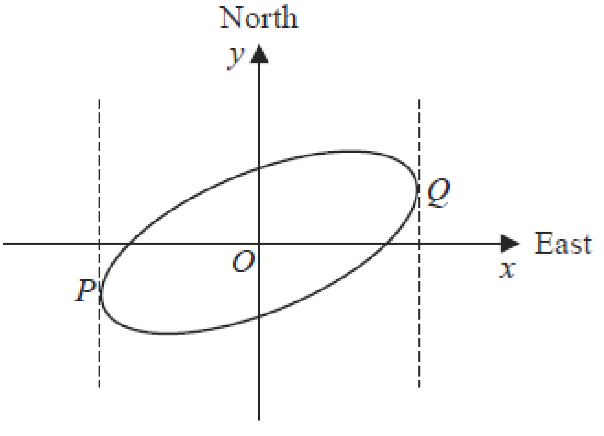


Figure 4

Figure 4 shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y - x}{3y - x}$

(4)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest west and furthest east of the origin O, as shown in Figure 4.

Using part (a),

(b) find the exact coordinates of the point P.

(5)

(c) Explain briefly how to find the coordinates of the point that is furthest north of the origin O. (You **do not** need to carry out this calculation).

(1)

10. The height above ground, H metres, of a passenger on a roller coaster can be modelled by the differential equation

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{H\cos(0.25t)}{40}$$

where t is the time, in seconds, from the start of the ride.

Given that the passenger is 5 m above the ground at the start of the ride,

(a) show that $H = 5\mathrm{e}^{0.1\sin(0.25t)}$

(5)

(b) State the maximum height of the passenger above the ground.

(1)

The passenger reaches the maximum height, for the second time, T seconds after the start of the ride.

(c) Find the value of T.

(2)

11. (a) Use binomial expansions to show that $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$

(6)

A student substitutes $x = \frac{1}{2}$ into both sides of the approximation shown in part (a) in an attempt to find an approximation to $\sqrt{6}$

(b) Give a reason why the student **should not** use $x = \frac{1}{2}$

(1)

(c) Substitute $x = \frac{1}{11}$ into

$$\sqrt{\frac{1+4x}{1-x}} = 1 + \frac{5}{2}x - \frac{5}{8}x^2$$

to obtain an approximation to $\sqrt{6}$. Give your answer as a fraction in its simplest form.

(3)

12. The value, £V, of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \qquad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

(a) (i) find p to 4 decimal places,

(ii) show that A is approximately 24 800

(4)

(b) With reference to the model, interpret

(i) the value of the constant A,

(ii) the value of the constant p.

(2)

Using the model,

(c) find the year during which the value of the car first exceeds £100 000

(4)

13. Show that

$$\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x = \frac{32}{15}\left(2+\sqrt{2}\right)$$

(7)

14. A curve C has parametric equations

$$x = 3 + 2\sin t, \qquad y = 4 + 2\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Show that all points on C satisfy $y = 6 - (x-3)^2$

(2)

(b) (i) Sketch the curve C.

(ii) Explain briefly why C does not include all points of $y = 6 - (x-3)^2, \quad x \in \mathbb{R}$

(3)

The line with equation $x + y = k$, where k is a constant, intersects C at two distinct points.

(c) State the range of values of k, writing your answer in set notation.

(5)

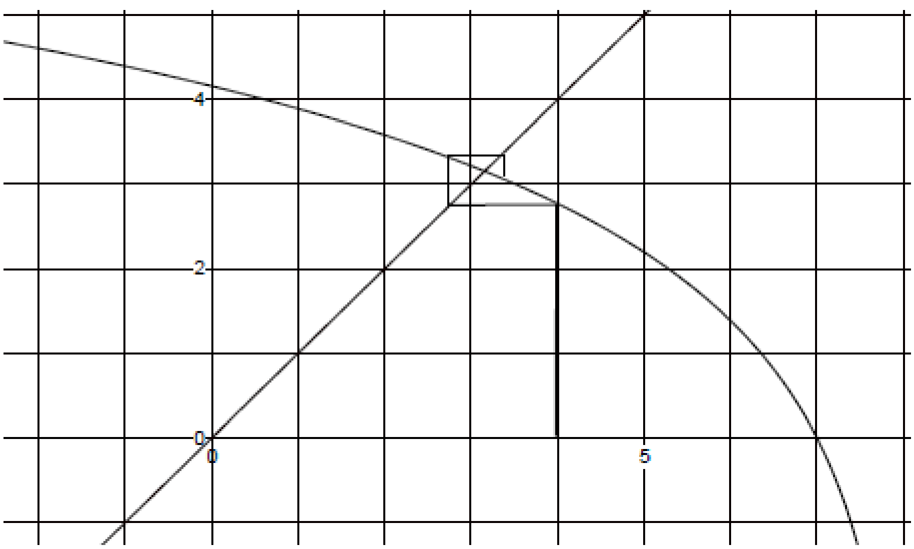
Question	Scheme	Marks	AOs
1	Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in $\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$	M1	1.1b
	Attempts both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2} \to \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify	M1	2.1
	$=\frac{4}{3}$ oe	A1	1.1b
		(3)	

Question	Scheme	Marks	AOs
2(a)	(i) $\frac{dy}{dx}=2x-2-12x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2}=2+6x^{-\frac{3}{2}}$	B1ft	1.1b
		(3)	
(b)	Substitutes $x=4$ into their $\frac{dy}{dx}=2\times 4-2-12\times 4^{-\frac{1}{2}}=\ldots$	M1	1.1b
	Shows $\frac{dy}{dx}=0$ and states "hence there is a stationary point" oe	A1	2.1
		(2)	
(c)	Substitutes $x=4$ into their $\frac{d^2y}{dx^2}=2+6\times 4^{-\frac{3}{2}}=(2.75)$	M1	1.1b
	$\frac{d^2y}{dx^2}=2.75>0$ and states "hence minimum"	A1ft	2.2a
		(2)	
			(7 marks)

Question	Scheme	Marks	AOs
3	States or uses $\frac{1}{2}r^2\theta=11$	B1	1.1b
	States or uses $2r+r\theta=4r\theta$	B1	1.1b
	Attempts to solve, full method $r=\ldots$	M1	3.1a
	$r=\sqrt{33}$	A1	1.1b
			[4]
			(4 marks)

Question	Scheme	Marks	AOs
4 (a)	Attempts $\mathrm{f}(3)=$ and $\mathrm{f}(4)=$ where $\mathrm{f}(x)=\pm\left(2\ln(8-x)-x\right)$	M1	2.1
	$\mathrm{f}(3)=\left(2\ln(5)-x\right)=(+)0.22$ and $\mathrm{f}(4)=\left(2\ln(4)-4\right)=-1.23$ Change of sign and function continuous in interval $[3,4]\Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	For annotating the graph by drawing a cobweb diagram starting at $x_1=4$ It should have at least two spirals	M1	2.4
	Deduces that the iteration formula **can be used** to find an approximation for α because **the cobweb spirals inwards** for the cobweb diagram	A1	2.2a
		(2)	

(4 marks)



5	$\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\dfrac{(2\sin\theta+2\cos\theta)3\cos\theta-3\sin\theta(2\cos\theta-2\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$	M1 A1	1.1b 1.1b
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator or uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\dfrac{\dots}{\dots\dots\dots C\sin\theta\cos\theta}$	M1	3.1a
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\dfrac{P}{Q+R\sin 2\theta}$	M1	2.1
	$\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\dfrac{3}{2+2\sin 2\theta}=\dfrac{3/2}{1+\sin 2\theta}$	A1	1.1b

(5 marks)

Question	Scheme	Marks	AOs
6 (a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient "$-\frac{1}{2}$" and point $(7,5)$ $y-5=-\frac{1}{2}(x-7)$	M1	1.1b
	Completes proof $2y+x=17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y+x=17$ and $y=2x+1$ simultaneously	M1	2.1
	$P=(3,7)$	A1	1.1b
	Length $PA=\sqrt{(3-7)^2+(7-5)^2}=(\sqrt{20})$	M1	1.1b
	Equation of C is $(x-7)^2+(y-5)^2=20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y=2x+k$ meets C using $\overrightarrow{OA}+\overrightarrow{PA}$	M1	3.1a
	Substitutes their $(11,3)$ in $y=2x+k$ to find k	M1	2.1
	$k=-19$	A1	1.1b
		(3)	
			(10 marks)
(c)	Attempts to find where $y=2x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $5x^2+(4k-34)x+k^2-10k+54=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=\ldots$	M1	2.1
	$k=-19$	A1	1.1b
		(3)	

Question	Scheme	Marks	AOs
7 (a)	$\int \frac{2}{(3x-k)}\,dx=\frac{2}{3}\ln(3x-k)$	M1 A1	1.1a 1.1b
	$\int_k^{3k} \frac{2}{(3x-k)}\,dx=\frac{2}{3}\ln(9k-k)-\frac{2}{3}\ln(3k-k)$	dM1	1.1b
	$=\frac{2}{3}\ln\left(\frac{8\cancel{k}}{2\cancel{k}}\right)=\frac{2}{3}\ln 4$ oe	A1	2.1
		(4)	
(b)	$\int \frac{2}{(2x-k)^2}\,dx=-\frac{1}{(2x-k)}$	M1	1.1b
	$\int_k^{2k} \frac{2}{(2x-k)^2}\,dx=-\frac{1}{(4k-k)}+\frac{1}{(2k-k)}$	dM1	1.1b
	$=\frac{2}{3k}\quad\left(\propto\frac{1}{k}\right)$	A1	2.1
		(3)	

Question	Scheme	Marks	AOs
8 (a)	$D=5+2\sin(30\times 6.5)°=$ awrt 4.48m with units	B1	3.4
		(1)	
(b)	$3.8=5+2\sin(30t)°\Rightarrow \sin(30t)°=-0.6$	M1 A1	1.1b 1.1b
	$t=10.77$	dM1	3.1a
	10:46 a.m. or 10:47 a.m.	A1	3.2a
		(4)	

Question	Scheme	Marks	AOs
9(a)	Either $3y^2\to Ay\frac{dy}{dx}$ or $2xy\to 2x\frac{dy}{dx}+2y$	M1	2.1
	$2x-2x\frac{dy}{dx}-2y+6y\frac{dy}{dx}=0$	A1	1.1b
	$(6y-2x)\frac{dy}{dx}=2y-2x$	M1	2.1
	$\frac{dy}{dx}=\frac{2y-2x}{6y-2x}=\frac{y-x}{3y-x}$ *	A1*	1.1b
		(4)	
(b)	$\left(\text{At } P \text{ and } Q\ \frac{dy}{dx}\to\infty\Rightarrow\right)$ Deduces that $3y-x=0$	M1	2.2a
	Solves $y=\frac{1}{3}x$ and $x^2-2xy+3y^2=50$ simultaneously	M1	3.1a
	$\Rightarrow x=(\pm)5\sqrt{3}$ OR $\Rightarrow y=(\pm)\frac{5}{3}\sqrt{3}$	A1	1.1b
	Using $y=\frac{1}{3}x\Rightarrow x=..$ AND $y=..$	dM1	1.1b
	$P=\left(-5\sqrt{3},-\frac{5}{3}\sqrt{3}\right)$	A1	2.2a
		(5)	
(c)	Explains that you need to solve $y=x$ and $x^2-2xy+3y^2=50$ simultaneously and choose the positive solution	B1ft	2.4
		(1)	

Question	Scheme	Marks	AOs
10(a)	$\frac{dH}{dt}=\frac{H\cos 0.25t}{40}\Rightarrow\int\frac{1}{H}dH=\int\frac{\cos 0.25t}{40}dt$	M1	3.1a
	$\ln H=\frac{1}{10}\sin 0.25t(+c)$	M1 A1	1.1b 1.1b
	Substitutes $t=0, H=5\Rightarrow c=\ln(5)$	dM1	3.4
	$\ln\left(\frac{H}{5}\right)=\frac{1}{10}\sin 0.25t\Rightarrow H=5e^{0.1\sin 0.25t}$ *	A1*	2.1
		(5)	
(b)	Max height = $5e^{0.1}=5.53$ m (Condone lack of units)	B1	3.4
		(1)	
(c)	Sets $0.25t=\frac{5\pi}{2}$	M1	3.1b
	31.4	A1	1.1b
		(2)	

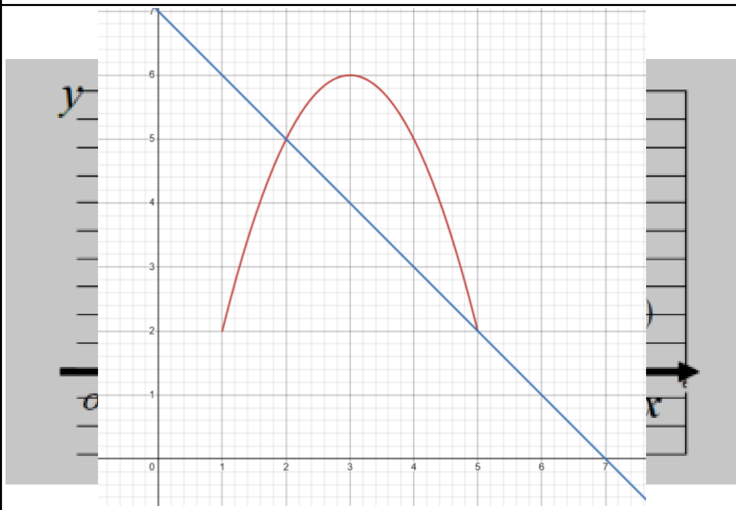
arks)

Question	Scheme	Marks	AOs
11 (a)	$\sqrt{\frac{1+4x}{1-x}}=(1+4x)^{0.5}\times(1-x)^{-0.5}$	B1	3.1a
	$(1+4x)^{0.5}=1+0.5\times(4x)+\frac{0.5\times-0.5}{2}\times(4x)^2$ $(1-x)^{-0.5}=1+(-0.5)(-x)+\frac{(-0.5)\times(-1.5)}{2}(-x)^2$ $(1+4x)^{0.5}=1+2x-2x^2$ and $(1-x)^{-0.5}=1+0.5x+0.375x^2$ oe	M1 M1 A1	1.1b 1.1b 1.1b
	$(1+4x)^{0.5}\times(1-x)^{-0.5}=(1+2x-2x^2......)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2...\right)$ $=1+\frac{1}{2}x+\frac{3}{8}x^2+2x+x^2-2x^2+...$ $=A+Bx+Cx^2$	dM1	2.1
	$=1+\frac{5}{2}x-\frac{5}{8}x^2......$ *	A1*	1.1b
		(6)	
(b)	Expression is valid $\lvert x\rvert<\frac{1}{4}$ Should not use $x=\frac{1}{2}$ as $\frac{1}{2}>\frac{1}{4}$	B1	2.3
		(1)	
(c)	Substitutes $x=\frac{1}{11}$ into $\sqrt{\frac{1+4x}{1-x}}\approx 1+\frac{5}{2}x-\frac{5}{8}x^2$	M1	1.1b
	$\sqrt{\frac{3}{2}}=\frac{1183}{968}$	A1	1.1b
	(so $\sqrt{6}$ is) $\frac{1183}{484}$ or $\frac{2904}{1183}$	A1	2.1
		(3)	

Question	Scheme	Marks	AOs
12 (a)	(i) Method to find p Eg. Divides $32000 = Ap^4$ by $50000 = Ap^{11}$ $p^7 = \frac{50000}{32000} \Rightarrow p = \sqrt[7]{\frac{50000}{32000}} = \ldots$	M1	3.1a
	$p = 1.0658$	A1	1.1b
	(ii) Substitutes their $p = 1.0658$ into either equation and finds A $A = \frac{32000}{'1.0658'^4}$ or $A = \frac{50000}{'1.0658'^{11}}$	M1	1.1b
	$A = 24795 \to 24805 \simeq 24\,800$ *	A1*	1.1b
		(4)	
(b)	A / (£) 24 800 is the value of the car on 1st January 2001	B1	3.4
	p / 1.0658 is the factor by which the value rises each year. Accept that the value rises by 6.6 % a year (ft on their p)		
(c)	Attempts $100000 = '24800' \times '1.0658'^t$		
	$'1.0658'^t = \frac{100000}{24800}$		
	$t = \log_{1.0658}\left(\frac{100000}{24800}\right)$		
	$t = 21.8$ or 21.9		
	cso 2022		

Question	Scheme for Substitution	
13	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,dx$ Award for • Using a valid substitution $u = \ldots$, changing the terms to u's • integrating and using appropriate limts .	
	Substitution $u = \sqrt{x+2} \Rightarrow \frac{dx}{du} = 2u$ oe	Substitution $u = x+2 \Rightarrow \frac{dx}{du} = 1$ oe
	$\int 2x\sqrt{x+2}\,dx$ $= \int A(u^2 \pm 2)u^2 du$	$\int 2x\sqrt{x+2}\,dx$ $= \int A(u \pm 2)\sqrt{u}\,du$
	$= Pu^5 \pm Qu^3$	$= Su^{\frac{5}{2}} \pm Tu^{\frac{3}{2}}$
	$= \frac{4}{5}u^5 - \frac{8}{3}u^3$	$= \frac{4}{5}u^{\frac{5}{2}} - \frac{8}{3}u^{\frac{3}{2}}$
	Uses limits 2 and $\sqrt{2}$ the correct way around	Uses limits 4 and 2 the correc[t] way around
	$= \frac{32}{15}(2+\sqrt{2})$ *	

Question Alt	Scheme for by parts	Marks	AOs
13	Chooses a suitable method for $\int_0^2 2x\sqrt{x+2}\,dx$ Award for • using by parts the correct way around • and using limits	M1	3.1a
	$\int (\sqrt{x+2})\,dx = \frac{2}{3}(x+2)^{\frac{3}{2}}$	B1	1.1b
	$\int 2x\sqrt{x+2}\,dx = Ax(x+2)^{\frac{3}{2}} - \int B(x+2)^{\frac{3}{2}}(dx)$	M1	1.1b
	$= Ax(x+2)^{\frac{3}{2}} - C(x+2)^{\frac{5}{2}}$	dM1	2.1
	$= \frac{4}{3}x(x+2)^{\frac{3}{2}} - \frac{8}{15}(x+2)^{\frac{5}{2}}$	A1	1.1b
	Uses limits 2 and 0 the correct way around	ddM1	1.1b
	$= \frac{32}{15}(2+\sqrt{2})$	A1*	2.1
		(7)	

Question	Scheme	Marks	AOs
14(a)	Attempts to use $\cos 2t = 1 - 2\sin^2 t \Rightarrow \dfrac{y-4}{2} = 1 - 2\left(\dfrac{x-3}{2}\right)^2$	M1	2.1
	$\Rightarrow y - 4 = 2 - 4 \times \dfrac{(x-3)^2}{4} \Rightarrow y = 6 - (x-3)^2$ *	A1*	1.1b
		(2)	
(b)	 $\cap$ shaped parabola	M1	1.1b
	Fully correct with 'ends' at (1,2) & (5,2)	A1	1.1b
	Suitable reason : Eg states as $x = 3 + 2\sin t$, $1 \leqslant x \leqslant 5$	B1	2.4
		(3)	
(c)	Either finds the lower value for $k = 7$ or deduces that $k < \dfrac{37}{4}$	B1	2.2a
	Finds where $x + y = k$ meets $y = 6 - (x-3)^2$ $\Rightarrow k - x = 6 - (x-3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
	Correct 3TQ in x $\quad x^2 - 7x + (k+3) = 0$ Or y $\quad y^2 + (7-2k)y + (k^2 - 6k + 3) = 0$	A1	1.1b
	Uses $b^2 - 4ac = 0 \Rightarrow \quad 49 - 4 \times 1 \times (k+3) = 0 \Rightarrow k = \left(\dfrac{37}{4}\right)$ or $(7-2k)^2 - 4 \times 1 \times (k^2 - 6k + 3) = 0 \Rightarrow k = \left(\dfrac{37}{4}\right)$	M1	2.1
	Range of values for $k = \left\{k : 7 \leqslant k < \dfrac{37}{4}\right\}$	A1	2.5
		(5)	

(10 marks)