0. AS Level (1)

3. A circle has equation

$$x^2 + y^2 - 10x + 16y = 80$$

(a) Find

(i) the coordinates of the centre of the circle,

(ii) the radius of the circle.

Given that P is the point on the circle that is furthest away from the origin O,

(b) find the exact length OP

5. The height, h metres, of a tree, t years after being planted, is modelled by the equation

$$h^2 = at + b \qquad 0 \leqslant t < 25$$

where a and b are constants.

Given that

- the height of the tree was 2.60 m, exactly 2 years after being planted
- the height of the tree was 5.10 m, exactly 10 years after being planted

(a) find a complete equation for the model, giving the values of a and b to 3 significant figures.

Given that the height of the tree was 7 m, exactly 20 years after being planted

(b) evaluate the model, giving reasons for your answer.

9. Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b-1}$$

(b) Write down the full restriction on the value of b, explaining the reason for this restriction.

2. (a) Sketch the curve with equation

$$y = 4^x$$

stating any points of intersection with the coordinate axes.

(b) Solve

$$4^x = 100$$

giving your answer to 2 decimal places.

4. Given that

$$y = 2x^2$$

use differentiation from first principles to show that

$$\frac{dy}{dx} = 4x$$

11. Prove, using algebra, that

$$n(n^2 + 5)$$

all $n \in \mathbb{N}$.

6.

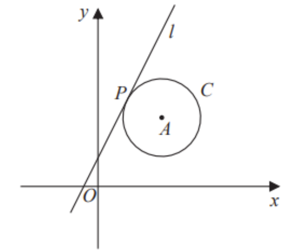


Figure 3

The circle C has centre A with coordinates $(7, 5)$.

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P, as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$ **(3)**

(b) Find an equation for C. **(4)**

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to C.

(c) Find the value of the constant k. **(3)**

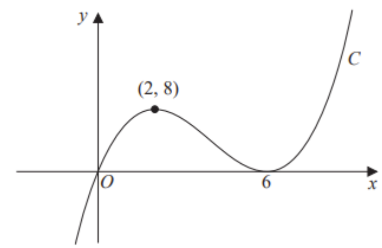


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ where $f(x)$ is a cubic expression in x.

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(a) Write down the set of values of x for which

$$f'(x) < 0$$

(1)

The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k, giving your answer in set notation. **(2)**

(c) Find the equation of C. You may leave your answer in factorised form. **(3)**

0. AS Level (2)

12. The value, £V, of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \qquad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

(a) (i) find p to 4 decimal places,

(ii) show that A is approximately 24 800

(4)

(b) With reference to the model, interpret

(i) the value of the constant A,

(ii) the value of the constant p.

(2)

Using the model,

(c) find the year during which the value of the car first exceeds £100 000

(4)

7. In a simple model, the value, £V, of a car depends on its age, t, in years.

The following information is available for car A

- its value when new is £20 000
- its value after one year is £16 000

(a) Use an exponential model to form, for car A, a possible equation linking V with t.

The value of car A is monitored over a 10-year period.
Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information.

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B.

8.

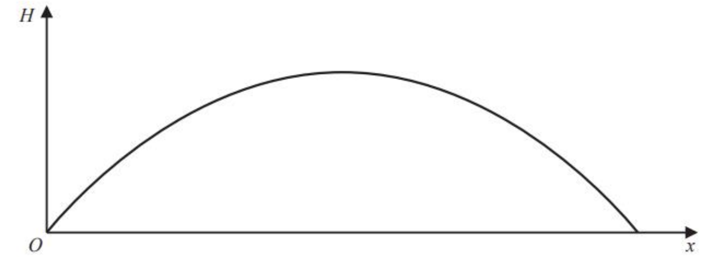


Figure 1

Figure 1 is a graph showing the trajectory of a rugby ball.

The height of the ball above the ground, H metres, has been plotted against the horizontal distance, x metres, measured from the point where the ball was kicked.

The ball travels in a vertical plane.

The ball reaches a maximum height of 12 metres and hits the ground at a point 40 metres from where it was kicked.

(a) Find a quadratic equation linking H with x that models this situation.

(3)

The ball passes over the horizontal bar of a set of rugby posts that is perpendicular to the path of the ball. The bar is 3 metres above the ground.

(b) Use your equation to find the greatest horizontal distance of the bar from O.

(3)

(c) Give one limitation of the model.

(1)

11. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds,

(2)

(b) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leqslant r \leqslant 20$, is

$$6 \times 1.05^{r-4}$$

(1)

(c) estimate the total time, in minutes and seconds, that she will take to complete the race.

(4)

1. Algebraic methods

2. $$f(x) = (x - 4)(x^2 - 3x + k) - 42 \quad \text{where } k \text{ is a constant}$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of k.

7. (i) Given that p and q are integers such that

$$pq \text{ is even}$$

use algebra to prove by contradiction that at least one of p or q is even.

(ii) Given that x and y are integers such that

- $x < 0$
- $(x + y)^2 < 9x^2 + y^2$

show that $y > 4x$

11.

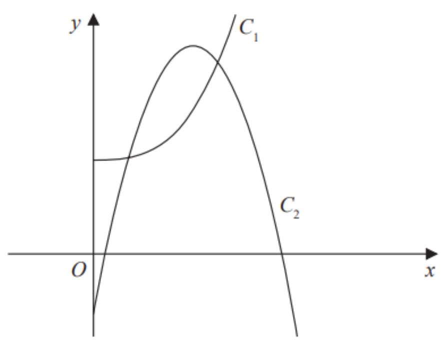


Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 2x^3 + 10 \qquad x > 0$$

and part of the curve C_2 with equation

$$y = 42x - 15x^2 - 7 \qquad x > 0$$

(a) Verify that the curves intersect at $x = \frac{1}{2}$

The curves intersect again at the point P

(b) Using algebra and showing all stages of working, find the exact x coordinate of P

6.

$$f(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$$

(a) (i) Calculate f(2)

(ii) Write $f(x)$ as a product of two algebraic factors.

Using the answer to (a)(ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0$$

(c) deduce the number of real solutions, for $7\pi \leqslant \theta < 10\pi$, to the equation

$$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0$$

11 .

$$\frac{1 + 11x - 6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{(x-3)} + \frac{C}{(1-2x)}$$

(a) Find the values of the constants A, B and C.

$$f(x) = \frac{1 + 11x - 6x^2}{(x-3)(1-2x)} \qquad x > 3$$

(b) Prove that $f(x)$ is a decreasing function.

2. Functions and graphs (1)

1. The point $P(-2, -5)$ lies on the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$

Find the point to which P is mapped, when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation

(a) $y = \mathrm{f}(x) + 2$

(b) $y = |\mathrm{f}(x)|$

(c) $y = 3\mathrm{f}(x-2) + 2$

2. $$\mathrm{f}(x) = (x-4)(x^2 - 3x + k) - 42 \quad \text{where } k \text{ is a constant}$$

Given that $(x+2)$ is a factor of $\mathrm{f}(x)$, find the value of k.

---

1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

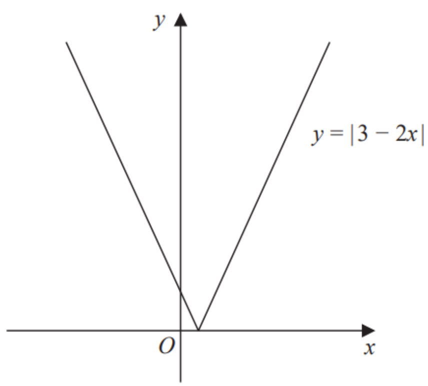


Figure 1

Figure 1 shows a sketch of the graph with equation $y = |3 - 2x|$

Solve

$$|3 - 2x| = 7 + x$$

10. The function f is defined by

$$\mathrm{f}(x) = \frac{8x+5}{2x+3} \qquad x > -\frac{3}{2}$$

(a) Find $\mathrm{f}^{-1}\left(\frac{3}{2}\right)$

(b) Show that

$$\mathrm{f}(x) = A + \frac{B}{2x+3}$$

where A and B are constants to be found.

The function g is defined by

$$\mathrm{g}(x) = 16 - x^2 \qquad 0 \leqslant x \leqslant 4$$

(c) State the range of g^{-1}

(d) Find the range of $\mathrm{f}\,\mathrm{g}^{-1}$

1.

$$\mathrm{g}(x) = \frac{2x+5}{x-3} \qquad x \geqslant 5$$

(a) Find gg(5).

(b) State the range of g.

(c) Find $\mathrm{g}^{-1}(x)$, stating its domain.

---

3. (a) "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."

Disprove this statement by means of a counter example.

(b) (i) Sketch the graph of $y = |x| + 3$

(ii) Explain why $|x| + 3 \geqslant |x + 3|$ for all real values of x.

2. Functions and graphs (2)

5. $$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write f(x) in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x - 2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

(ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

10. (i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4)

(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$."
State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

12. $$f(x) = 10e^{-0.25x} \sin x, \qquad x \geqslant 0$$

(a) Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

(4)

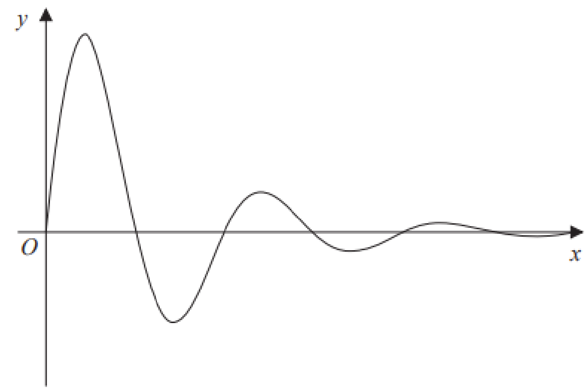


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$.

(b) Sketch the graph of H against t where

$$H(t) = |10e^{-0.25t} \sin t| \qquad t \geqslant 0$$

showing the long-term behaviour of this curve.

(2)

The function $H(t)$ is used to model the height, in metres, of a ball above the ground t seconds after it has been kicked.

Using this model, find

(c) the maximum height of the ball above the ground between the first and second bounce.

(3)

(d) Explain why this model should not be used to predict the time of each bounce.

(1)

3. Sequences and series

13. (i) In an arithmetic series, the first term is a and the common difference is d.

Show that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

(ii) James saves money over a number of weeks to buy a printer that costs £64

He saves £10 in week 1, £9.20 in week 2, £8.40 in week 3 and so on, so that the weekly amounts he saves form an arithmetic sequence.

Given that James takes n weeks to save exactly £64

(a) show that

$$n^2 - 26n + 160 = 0$$

(b) Solve the equation

$$n^2 - 26n + 160 = 0$$

(c) Hence state the number of weeks James takes to save enough money to buy the printer, giving a brief reason for your answer.

3. A sequence of terms $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3$$
$$a_{n+1} = 8 - a_n$$

(a) (i) Show that this sequence is periodic.

(ii) State the order of this periodic sequence.

(b) Find the value of

$$\sum_{n=1}^{85} a_n$$

4. (i) Show that $\sum_{r=1}^{16}(3 + 5r + 2^r) = 131\,798$

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} u_r$

4. Binomial expansion

7. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\sqrt{4 - 9x}$$

writing each term in simplest form.

A student uses this expansion with $x = \frac{1}{9}$ to find an approximation for $\sqrt{3}$

Using the answer to part (a) and without doing any calculations,

(b) state whether this approximation will be an overestimate or an underestimate of $\sqrt{3}$ giving a brief reason for your answer.

11. (a) Use binomial expansions to show that $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$ **(6)**

A student substitutes $x = \frac{1}{2}$ into both sides of the approximation shown in part (a) in an attempt to find an approximation to $\sqrt{6}$

(b) Give a reason why the student **should not** use $x = \frac{1}{2}$ **(1)**

(c) Substitute $x = \frac{1}{11}$ into

$$\sqrt{\frac{1+4x}{1-x}} = 1 + \frac{5}{2}x - \frac{5}{8}x^2$$

to obtain an approximation to $\sqrt{6}$. Give your answer as a fraction in its simplest form. **(3)**

10. Numerical methods

5. The table below shows corresponding values of x and y for $y = \log_3 2x$

The values of y are given to 2 decimal places as appropriate.

x	3	4.5	6	7.5	9
y	1.63	2	2.26	2.46	2.63

(a) Using the trapezium rule with all the values of y in the table, find an estimate for

$$\int_3^9 \log_3 2x \, dx$$

Using your answer to part (a) and making your method clear, estimate

(b) (i) $\int_3^9 \log_3 (2x)^{10} \, dx$

(ii) $\int_3^9 \log_3 18x \, dx$

6.

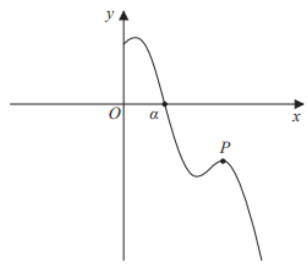


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = 8\sin\left(\frac{1}{2}x\right) - 3x + 9 \qquad x > 0$$

and x is measured in radians.

The point P, shown in Figure 2, is a local maximum point on the curve.

Using calculus and the sketch in Figure 2,

(a) find the x coordinate of P, giving your answer to 3 significant figures.

The curve crosses the x-axis at $x = \alpha$, as shown in Figure 2.

Given that, to 3 decimal places, $f(4) = 4.274$ and $f(5) = -1.212$

(b) explain why α must lie in the interval $[4, 5]$

(c) Taking $x_0 = 5$ as a first approximation to α, apply the Newton-Raphson method once to $f(x)$ to obtain a second approximation to α.

Show your method and give your answer to 3 significant figures.

4. The curve with equation $y = 2\ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$

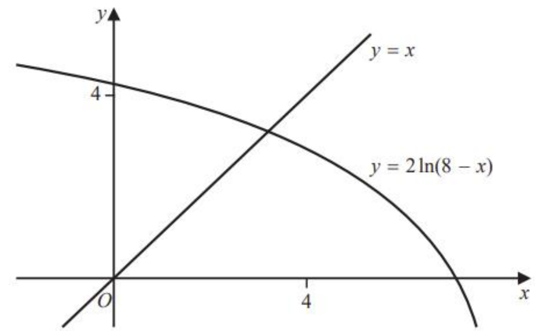


Figure 2

Figure 2 shows the graph of $y = 2\ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2\ln(8 - x_n), \quad n \in \mathbb{N}$$

in an attempt to find an approximation for α.

Using the graph and starting with $x_1 = 4$

(b) determine whether or not this iteration formula can be used to find an approximation for α, justifying your answer.

5. The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root.

(a) Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$$

Using the formula given in part (a) with $x_1 = 1$

(b) find the values of x_2 and x_3

(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$

12. Vectors

9.

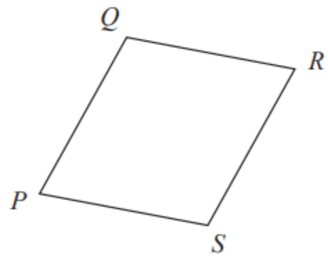


Figure 3

Figure 3 shows a sketch of a parallelogram *PQRS*.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$
- $\overrightarrow{QR} = 5\mathbf{i} - 2\mathbf{k}$

(a) show that parallelogram *PQRS* is a rhombus.

(b) Find the exact area of the rhombus *PQRS*.

2. Relative to a fixed origin O,

the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$,

the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

(a) Find the position vector of D.

Given $|\overrightarrow{AC}| = 4$

(b) find the value of a.

13. Relative to a fixed origin O

- the point A has position vector $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $4\mathbf{j} + 6\mathbf{k}$
- the point C has position vector $-16\mathbf{i} + p\mathbf{j} + 10\mathbf{k}$

where p is a constant.

Given that A, B and C lie on a straight line,

(a) find the value of p.

The line segment OB is extended to a point D so that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

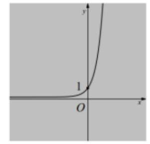
(b) Find $\left|\overrightarrow{OD}\right|$, writing your answer as a fully simplified surd.

0. AS Level (1)

Question	Scheme	Marks	AOs
3 (a)	(i) $x^2+y^2-10x+16y=80 \Rightarrow (x-5)^2+(y+8)^2=...$	M1	1.1b
	Centre $(5,-8)$	A1	1.1b
	(ii) Radius 13	A1	1.1b
		(3)	
(b)	Attempts $\sqrt{"5"^2+"8"^2}+"13"$	M1	3.1a
	$13+\sqrt{89}$ but ft on their centre and radius	A1ft	1.1b
		(2)	
			(5 marks)

Question	Scheme	Marks	AOs
5 (a)	Attempts to use $h^2=at+b$ with either $t=2, h=2.6$ or $t=10, h=5.1$	M1	3.1b
	Correct equations $2a+b=6.76$, $10a+b=26.01$	A1	1.1b
	Solves simultaneously to find values for a and b	dM1	1.1b
	$h^2=2.41t+1.95$ cao	A1	3.3
		(4)	
(b)	Substitutes $t=20$ into their $h^2=2.41t+1.95$ and finds h or h^2 Or substitutes $h=7$ into their $h^2=2.41t+1.95$ and finds t	M1	3.4
	Compares the model with the true values and concludes "good model" with a minimal reason E.g. I Finds $h=7.08$ (m) and states that it is a good model as 7.08 (m) is close to 7 (m) E.g II Finds $t=19.5$ years and states that the model is accurate as 19.5 (years) ≈ 20 (years)	A1	3.5a
		(2)	
			(6 marks)

9 (a)	States $\log a-\log b=\log\frac{a}{b}$	B1
	Proceeds from $\frac{a}{b}=a-b \rightarrow ...... \rightarrow ab-a=b^2$	M1
	$ab-a=b^2 \rightarrow a(b-1)=b^2 \Rightarrow a=\frac{b^2}{b-1}$ *	A1*
		(3)
(b)	States either $b>1$ or $b\neq 1$ with reason $\frac{b^2}{b-1}$ is not defined at $b=1$ oe	B1
	States $b>1$ and explains that as $a>0 \Rightarrow \frac{b^2}{b-1}>0 \Rightarrow b>1$	B1
		(2)

2(a)		Correct shape or correct intercept – see notes	B1
		Fully correct – see notes	B1
			(2)
(b)	$4^x=100 \Rightarrow x=\log_4 100$ or e.g. $x\log 4=\log 100 \Rightarrow x=\frac{\log 100}{\log 4}$		M1
	$\Rightarrow (x=)$ awrt 3.32		A1
			(2)

4	$\frac{2(x+h)^2-2x^2}{h}=...$	M1
	$\frac{2(x+h)^2-2x^2}{h}=\frac{4xh+2h^2}{h}$	A1
	$\frac{dy}{dx}=\lim_{h\to 0}\frac{4xh+2h^2}{h}=\lim_{h\to 0}(4x+2h)=4x$ *	A1*
		(3)

$n(n^2+5)$	
Attempts even **or** odd numbers Sets $n=2k$ **or** $n=2k\pm 1$ oe and attempts $n(n^2+5)$	M1

6 (a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1
	Finding the equation of a line with gradient "$-\frac{1}{2}$" and point $(7,5)$ $y-5=-\frac{1}{2}(x-7)$	M1
	Completes proof $2y+x=17$ *	A1*
		(3)
(b)	Solves $2y+x=17$ and $y=2x+1$ simultaneously	M1
	$P=(3,7)$	A1
	Length $PA=\sqrt{(3-7)^2+(7-5)^2}=(\sqrt{20})$	M1
	Equation of C is $(x-7)^2+(y-5)^2=20$	A1
		(4)
(c)	Attempts to find where $y=2x+k$ meets C using $\overrightarrow{OA}+\overrightarrow{PA}$	M1
	Substitutes their $(11,3)$ in $y=2x+k$ to find k	M1
	$k=-19$	A1
		(3)
(c)	Attempts to find where $y=2x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $5x^2+(4k-34)x+k^2-10k+54=0$	M1
	Uses $b^2-4ac=0$ oe and proceeds to $k=...$	M1
	$k=-19$	A1
		(3)

Question	Scheme	Marks	AOs
6 (a)	$2<x<6$	B1	1.1b
		(1)	
(b)	States either $k>8$ or $k<0$	M1	3.1a
	States e.g. $\{k: k>8\} \cup \{k: k<0\}$ Error: only k<0	A1	2.5
		(2)	
(c)	**Please see notes for alternatives**		
	States $y=ax(x-6)^2$ or $f(x)=ax(x-6)^2$	M1	1.1b
	Substitutes $(2,8)$ into $y=ax(x-6)^2$ and attempts to find a	dM1	3.1a
	$y=\frac{1}{4}x(x-6)^2$ or $f(x)=\frac{1}{4}x(x-6)^2$ o.e	A1	2.1
		(3)	
			(6 marks)

0. AS Level (2)

12 (a)	(i) Method to find p Eg. Divides $32000 = Ap^4$ by $50000 = Ap^{11}$ $p^7 = \frac{50000}{32000} \Rightarrow p = \sqrt[7]{\frac{50000}{32000}} = ...$	M1
	$p = 1.0658$	A1
	(ii) Substitutes their $p = 1.0658$ into either equation and finds A $A = \frac{32000}{'1.0658'^4}$ or $A = \frac{50000}{'1.0658'^{11}}$	M1
	$A = 24795 \to 24805 \simeq 24\,800$ *	A1*
		(4)
(b)	A / (£)24 800 is the value of the car on 1st January 2001	B1
	p /1.0658 is the factor by which the value rises each year. Accept that the value rises by 6.6 % a year (ft on their p)	B1
		(2)
(c)	Attempts $100000 = '24800' \times '1.0658'^t$	
	$'1.0658'^t = \frac{100000}{24800}$	M1
	$t = \log_{1.0658}\left(\frac{100000}{24800}\right)$	dM1
	$t = 21.8$ or 21.9	A1
	cso 2022	A1
		(4)

7 (a)	Uses a model $V = Ae^{\pm kt}$ oe (See next page for other suitable models)	M1
	Eg. Substitutes $t = 0, V = 20\,000 \Rightarrow A = 20000$	M1
	Eg. Substitutes $t = 1, V = 16000 \Rightarrow 16000 = 20000e^{-1k} \Rightarrow k = ..$	dM1
	$V = 20000e^{-0.223t}$	A1
		(4)
(b)	Substitutes $t = 10$ in their $V = 20000e^{-0.223t} \Rightarrow V = (£\,2150)$	M1
	Eg. The model is reliable as £2150 ≈ £2000	A1
		(2)
(c)	Make the "-0.223" less negative. Alt: Adapt model to for example $V = 18000e^{-0.223t} + 2000$	B1ft
		(1)

8 (a) Way 1	$H = Ax(40-x)$ {or $H = Ax(x-40)$ }	M1	3.3
	$x = 20, H = 12 \Rightarrow 12 = A(20)(40-20) \Rightarrow A = \frac{3}{100}$	dM1	3.1b
	$H = \frac{3}{100}x(40-x)$ or $H = -\frac{3}{100}x(x-40)$	A1	1.1b
		(3)	
(a) Way 2	$H = 12 - \lambda(x-20)^2$ {or $H = 12 + \lambda(x-20)^2$ }	M1	3.3
	$x = 40, H = 0 \Rightarrow 0 = 12 - \lambda(40-20)^2 \Rightarrow \lambda = \frac{3}{100}$	dM1	3.1b
	$H = 12 - \frac{3}{100}(x-20)^2$	A1	1.1b
		(3)	
(a) Way 3	$H = ax^2 + bx + c$ (or deduces $H = ax^2 + bx$) **Both** $x = 0, H = 0 \Rightarrow 0 = 0 + 0 + c \Rightarrow c = 0$ **and either** $x = 40, H = 0 \Rightarrow 0 = 1600a + 40b$ **or** $x = 20, H = 12 \Rightarrow 12 = 400a + 20b$ **or** $\frac{-b}{2a} = 20$ $\{\Rightarrow b = -40a\}$	M1	3.3
	$b = -40a \Rightarrow 12 = 400a + 20(-40a) \Rightarrow a = -0.03$ so $b = -40(-0.03) = 1.2$	dM1	3.1b
	$H = -0.03x^2 + 1.2x$	A1	1.1b
		(3)	
(b)	$\{H = 3 \Rightarrow\}$ $3 = \frac{3}{100}x(40-x) \Rightarrow x^2 - 40x + 100 = 0$ **or** $\{H = 3 \Rightarrow\}$ $3 = 12 - \frac{3}{100}(x-20)^2 \Rightarrow (x-20)^2 = 300$	M1	3.4
	e.g. $x = \frac{40 \pm \sqrt{1600 - 4(1)(100)}}{2(1)}$ or $x = 20 \pm \sqrt{300}$	dM1	1.1b
	$\{$chooses $20 + \sqrt{300} \Rightarrow\}$ greatest distance = awrt 37.3 m	A1	3.2a
		(3)	
(c)	Gives a limitation of the model. Accept e.g. • the ground is horizontal • the ball needs to be kicked from the ground • the ball is modelled as a particle • the horizontal bar needs to be modelled as a line • there is no wind or air resistance on the ball • there is no spin on the ball • no obstacles in the trajectory (or path) of the ball • the trajectory of the ball is a perfect parabola	B1	3.5b
		(1)	
			(7 marks)

11 (a)	Total time for 6 km = 24 minutes + $6\times1.05 + 6\times1.05^2$ minutes	M1
	= 36.915 minutes = 36 minutes 55 seconds *	A1*
		(2)
(b)	5^{th} km is $6\times1.05 = 6\times1.05^1$ 6^{th} km is $6\times1.05\times1.05 = 6\times1.05^2$ 7^{th} km is $6\times1.05\times1.05\times1.05 = 6\times1.05^3$ Hence the time for the r^{th} km is $6\times1.05^{r-4}$	B1
		(1)
(c)	Attempts the total time for the race = Eg. 24 minutes + $\sum_{r=5}^{r=20} 6\times1.05^{r-4}$ minutes	M1
	Uses the series formula to find an allowable sum Eg. Time for 5^{th} to 20^{th} km $= \frac{6.3(1.05^{16}-1)}{1.05-1} = (149.04)$	M1
	Correct calculation that leads to the **total time** Eg. Total time $= 24 + \frac{6.3(1.05^{16}-1)}{1.05-1}$	A1
	Total time = awrt 173 minutes and 3 seconds	A1
		(4)

1. Algebraic methods

Sets $f(-2)=0 \Rightarrow (-2-4)((-2)^2-3\times-2+k)-42=0$
$-6(k+10)=42 \Rightarrow k=...$
$k=-17$

7 (i)	For setting up the contradiction: There exists integers p and q such that pq is even and both p and q are odd	B1	2.5
	For example, sets $p=2m+1$ and $q=2n+1$ and then attempts $pq=(2m+1)(2n+1)=...$	M1	1.1b
	Obtains $pq=(2m+1)(2n+1)=4mn+2m+2n+1$ $=2(2mn+m+n)+1$ States that this is odd, giving a contradiction so " if pq is even, then at least one of p and q is even" *	A1*	2.1
		(3)	
(ii)			
	$(x+y)^2<9x^2+y^2 \Rightarrow 2xy<8x^2$	M1	2.2a
	States that as $x<0 \Rightarrow 2y>8x$ $\Rightarrow y>4x$ *	A1*	2.1
		(2)	
			(5 marks)

11 (a)	Substitutes $x=\frac{1}{2}$ into $y=2x^3+10$ **and** $y=42x-15x^2-7$ and finds the y values for both	M1
	Achieves $\frac{41}{4}$ o.e. for both and makes a valid conclusion. *	A1*
		(2)
(b)	Sets $42x-15x^2-7=2x^3+10 \Rightarrow 2x^3+15x^2-42x+17=0$	M1
	Deduces that $(2x-1)$ is a factor and attempts to divide	dM1
	$2x^3+15x^2-42x+17=(2x-1)(x^2+8x-17)$	A1
	Solves their $x^2+8x-17=0$ using suitable method	M1
	Deduces $x=-4+\sqrt{33}$ (see note)	A1
		(5)

6	(a) $f(x)=-3x^3+8x^2-9x+10, \quad x\in\mathbb{R}$	
(a)	(i) $\{f(2)=-24+32-18+10 \Rightarrow\}\ f(2)=0$	B1
	(ii) $\{f(x)=\}\ (x-2)(-3x^2+2x-5)$ or $(2-x)(3x^2-2x+5)$	M1 A1
		(3)
(b)	$-3y^6+8y^4-9y^2+10=0 \Rightarrow (y^2-2)(-3y^4+2y^2-5)=0$	
	Gives a partial explanation by • explaining that $-3y^4+2y^2-5=0$ has no {real} solutions with a reason, e.g. $b^2-4ac=(2)^2-4(-3)(-5)=-56<0$ • or stating that $y^2=2$ has 2 {real} solutions or $y=\pm\sqrt{2}$ {only}	M1
	Complete proof that the given equation has exactly two {real} solutions	A1
		(2)
(c)	$3\tan^3\theta-8\tan^2\theta+9\tan\theta-10=0;\ 7\pi\le\theta<10\pi$	
	{Deduces that} there are 3 solutions	B1
		(1)

11	$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A+\frac{B}{(x-3)}+\frac{C}{(1-2x)}$	
(a) Way 1	$1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1
	$A=3$	B1
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1
	$B=4$ and $C=-2$ which have been found using a correct identity	A1
		(4)
(a) Way 2	{long division gives} $\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3+\frac{-10x+10}{(x-3)(1-2x)}$	
	$-10x+10 \equiv B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1
	$A=3$	B1
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1
	$B=4$ and $C=-2$ which have been found using $-10x+10 \equiv B(1-2x)+C(x-3)$	A1
		(4)
(b)	$f(x)=3+\frac{4}{(x-3)}-\frac{2}{(1-2x)}$ $\{=3+4(x-3)^{-1}-2(1-2x)^{-1}\}$; $x>3$	
	$f'(x)=-4(x-3)^{-2}-4(1-2x)^{-2}$ $\left\{=-\frac{4}{(x-3)^2}-\frac{4}{(1-2x)^2}\right\}$	M1 A1ft
	Correct $f'(x)$ **and** as $(x-3)^2>0$ and $(1-2x)^2>0$, then $f'(x)=-(+\text{ve})-(+\text{ve})<0$, so $f(x)$ is a decreasing function	A1
		(3)

2. Functions and graphs (1)

Question	
1 (a)	$(-2, -3)$
(b)	$(-2, 5)$
(c)	Either $x = 0$ or $y = -13$
	$(0, -13)$

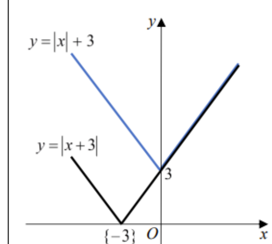
Sets $f(-2) = 0 \Rightarrow (-2-4)\left((-2)^2 - 3\times -2 + k\right) - 42 = 0$	M1
$-6(k+10) = 42 \Rightarrow k = ...$	M1
$k = -17$	A1
	(3)

For an attempt to solve **Either** $3-2x = 7+x \Rightarrow x = ...$ **or** $2x-3 = 7+x \Rightarrow x = ...$
Either $x = -\frac{4}{3}$ **or** $x = 10$
For an attempt to solve **Both** $3-2x = 7+x \Rightarrow x = ...$ **and** $2x-3 = 7+x \Rightarrow x = ...$
For **both** $x = -\frac{4}{3}$ **and** $x = 10$ with no extra solutions
Alternative by squaring:
$(3-2x)^2 = (7+x)^2 \Rightarrow 9 - 12x + 4x^2 = 49 + 14x + x^2$
$3x^2 - 26x - 40 = 0$
$3x^2 - 26x - 40 = 0 \Rightarrow x = ...$
For **both** $x = -\frac{4}{3}$ **and** $x = 10$ with no extra solutions

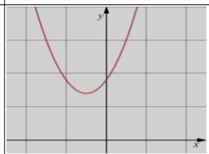
Doesn't work, eg y=|4-3x| and y=-x-4

10(a)	Attempts to solve $\frac{3}{2} = \frac{8x+5}{2x+3} \Rightarrow x = ...$ Or substitutes $x = \frac{3}{2}$ into $\frac{5-3x}{2x-8}$	M1	3.1a
	$\left(f^{-1}\left(\frac{3}{2}\right) = \right) - \frac{1}{10}$	A1	1.1b
		(2)	
(b)	$\left(\frac{8x+5}{2x+3} = \right) 4 \pm \frac{...}{2x+3}$	M1	1.1b
	$\left(\frac{8x+5}{2x+3} = \right) 4 - \frac{7}{2x+3}$	A1	2.1
		(2)	
(c)	$0 \leqslant g^{-1}(x) \leqslant 4$	B1	2.2a
		(1)	
(d)	Attempts either boundary $f(0) = \frac{8\times 0+5}{2\times 0+3}$ **or** $f(4) = \frac{8\times 4+5}{2\times 4+3}$	M1	3.1a
	Attempts both boundaries $f(0) = \frac{8\times 0+5}{2\times 0+3}$ **and** $f(4) = \frac{8\times 4+5}{2\times 4+3}$	dM1	1.1b
	Range $\frac{5}{3} \leqslant fg^{-1}(x) \leqslant \frac{37}{11}$	A1	2.1
		(3)	
	Alternative by attempting $fg^{-1}(x)$		
	$g^{-1}(x) = \sqrt{16-x} \Rightarrow fg^{-1}(x) = \frac{8\sqrt{16-x}+5}{2\sqrt{16-x}+3}$ $fg^{-1}(0) = \frac{8\times 4+5}{2\times 4+3}$ or $fg^{-1}(16) = \frac{8\times 0+5}{2\times 0+3}$	M1	3.1a
	$fg^{-1}(0) = \frac{8\times 4+5}{2\times 4+3}$ **and** $fg^{-1}(16) = \frac{8\times 0+5}{2\times 0+3}$	dM1	1.1b
	Range $\frac{5}{3} \leqslant fg^{-1}(x) \leqslant \frac{37}{11}$	A1	2.1
		(3)	
		(8 marks)	

1	$g(x) = \frac{2x+5}{x-3},\ x \geq 5$	
(a) **Way 1**	$g(5) = \frac{2(5)+5}{5-3} = 7.5 \Rightarrow gg(5) = \frac{2("7.5")+5}{"7.5"-3}$	M1
	$gg(5) = \frac{40}{9}$ $\left(\text{or } 4\frac{4}{9} \text{ or } 4.\dot{4}\right)$	A1
		(2)
(a) **Way 2**	$gg(x) = \frac{2\left(\frac{2x+5}{x-3}\right)+5}{\left(\frac{2x+5}{x-3}\right)-3} \Rightarrow gg(5) = \frac{2\left(\frac{2(5)+5}{(5)-3}\right)+5}{\left(\frac{2(5)+5}{(5)-3}\right)-3}$	M1
	$gg(5) = \frac{40}{9}$ $\left(\text{or } 4\frac{4}{9} \text{ or } 4.\dot{4}\right)$	A1
		(2)
(b)	{Range:} $2 < y \leq \frac{15}{2}$	B1
		(1)
(c) **Way 1**	$y = \frac{2x+5}{x-3} \Rightarrow yx - 3y = 2x + 5 \Rightarrow yx - 2x = 3y + 5$	M1
	$x(y-2) = 3y+5 \Rightarrow x = \frac{3y+5}{y-2}$ $\left\{\text{or } y = \frac{3x+5}{x-2}\right\}$	M1
	$g^{-1}(x) = \frac{3x+5}{x-2},\quad 2 < x \leq \frac{15}{2}$	A1ft
		(3)
(c) **Way 2**	$y = \frac{2x-6+11}{x-3} \Rightarrow y = 2 + \frac{11}{x-3} \Rightarrow y - 2 = \frac{11}{x-3}$	M1
	$x - 3 = \frac{11}{y-2} \Rightarrow x = \frac{11}{y-2} + 3$ $\left\{\text{or } y = \frac{11}{x-2} + 3\right\}$	M1
	$g^{-1}(x) = \frac{11}{x-2} + 3,\quad 2 < x \leq \frac{15}{2}$	A1ft
		(3)

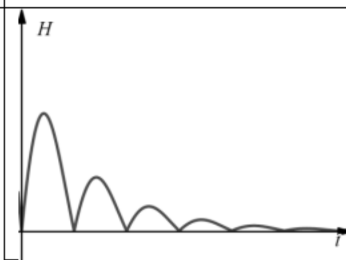
Question	Scheme		Marks
3	Statement: "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."		
(a)	E.g. $m = \sqrt{3},\ n = \sqrt{12}$		M1
	$\{mn =\}\ (\sqrt{3})(\sqrt{12}) = 6$ $\Rightarrow$ statement untrue **or** 6 is not irrational or 6 is rational		A1
			(2)
(b)(i), **(ii) Way 1**		V shaped graph {reasonably} symmetrical about the y-axis with vertical interpret $(0, 3)$ or 3 stated or marked on the positive y-axis	B1
		Superimposes the graph of $y = \lvert x+3 \rvert$ on top of the graph of $y = \lvert x \rvert + 3$	M1
	the graph of $y = \lvert x \rvert + 3$ is either the same or above the graph of $y = \lvert x+3 \rvert$ {for corresponding values of x} **or** when $x \geq 0$, both graphs are equal (or the same) when $x < 0$, the graph of $y = \lvert x \rvert + 3$ is above the graph of $y = \lvert x+3 \rvert$		A1
			(3)
(b)(ii) **Way 2**	<u>**Reason 1**</u> When $x \geq 0$, $\lvert x \rvert + 3 = \lvert x+3 \rvert$ <u>**Reason 2**</u> When $x < 0$, $\lvert x \rvert + 3 > \lvert x+3 \rvert$	Any one of Reason 1 or Reason 2	M1
		Both Reason 1 and Reason 2	A1

2. Functions and graphs (2)

5 (a)	$2x^2+4x+9=2(x\pm k)^2\pm....$ $\quad a=2$	B1
	Full method $2x^2+4x+9=2(x+1)^2\pm...$ $\quad a=2\ \&b=1$	M1
	$2x^2+4x+9=2(x+1)^2+7$	A1
		(3)
(b)	 U shaped curve any position but not through (0,0)	B1
	y - intercept at $(0,9)$	B1
	Minimum at $(-1,7)$	B1ft
		(3)
(c)	(i) Deduces translation with one correct aspect.	M1
	Translate $\begin{pmatrix}2\\-4\end{pmatrix}$	A1
	(ii) $\mathrm{h}(x)=\dfrac{21}{"2(x+1)^2+7"}\Rightarrow$ (maximum) value $\dfrac{21}{"7"}(=3)$	M1
	$0<\mathrm{h}(x)\leqslant 3$	A1ft
		(4)

For $n=2m,\quad n^2+2=4m^2+2$	M1
Concludes that this number is not divisible by 4 (as the explanation is trivial)	A1
For $n=2m+1,\quad n^2+2=(2m+1)^2+2=...$ $\qquad$ FYI $\left(4m^2+4m+3\right)$	dM1
Correct working and concludes that this is a number in the 4 times table add 3 so cannot be divisible by 4 or writes $4\left(m^2+m\right)+3$ ....AND states ......hence true for all	A1*
	(4)

SOMETIMES TRUE and chooses any number $x:9.25<x<9.5$ and shows false $\qquad$ Eg $x=9.4\quad \lvert 3x-28\rvert=0.2$ and $x-9=0.4$ ×	M1
Then chooses a number where it is true Eg $x=12\quad \lvert 3x-28\rvert=8\quad x-9=3$ ✓	A1
	(2)

12 (a)	$\mathrm{f}(x)=10\mathrm{e}^{-0.25x}\sin x$	
	$\Rightarrow \mathrm{f}'(x)=-2.5\mathrm{e}^{-0.25x}\sin x+10\mathrm{e}^{-0.25x}\cos x$ oe	M1 A1
	$\mathrm{f}'(x)=0\Rightarrow -2.5\cancel{\mathrm{e}^{-0.25x}}\sin x+10\cancel{\mathrm{e}^{-0.25x}}\cos x=0$	M1
	$\dfrac{\sin x}{\cos x}=\dfrac{10}{2.5}\Rightarrow \tan x=4$*	A1*
		(4)
(b)	 "Correct" shape for 2 loops	M1
	Fully correct with decreasing heights	A1
		(2)
(c)	Solves $\tan x=4$ and substitutes answer into $H(t)$	M1
	$H(4.47)=\lvert 10\mathrm{e}^{-0.25\times 4.47}\sin 4.47\rvert$	M1
	awrt 3.18 (metres)	A1
		(3)
(d)	The **times between** each bounce should not stay the same when the heights of each bounce is getting smaller	B1
		(1)

3. Sequences and series

13 (i)	States that $S = a+(a+d)+ \ldots\ldots\ldots + (a+(n-1)d)$	B1
	$S = a + \quad . \quad (a+d)+ \quad (a+(n-1)d)$ $S = (a+(n-1)d)+(a+(n-2)d)+\ldots\ldots\ldots+a$	M1
	Reaches $2S = n\times(2a+(n-1)d)$ And so proves that $S=\frac{n}{2}[2a+(n-1)d]$ *	A1*
		(3)
(ii)	(a) $S = 10+9.20+8.40+\ldots$	
	$64=\frac{n}{2}(20-0.8(n-1))$ o.e	M1
	$128 = 20n-0.8n^2+0.8n$ $0.8n^2-20.8n+128=0$ $n^2-26n+160=0$ *	A1*
		(2)
	(b) $n=10,16$	B1
		(1)
	(c) 10 weeks with a minimal correct reason. E.g. • He has saved up the amount by 10 weeks so he would not save for another 6 weeks • You would choose the smaller number • He starts saving negative amounts (in week 14) so 16 does not make sense	B1
		(1)

3(a)(i)	$a_1=3,\ a_2=5,\ a_3=3\ \ldots$	B1
(ii)	2	B1
		(2)
(b)	$\sum_{n=1}^{85} a_n = 42\times(3+5)+3$ o.e.	M1
	$=339$	A1
		(2)

4	(i) $\sum_{r=1}^{16}(3+5r+2^r)=131\,798$; (ii) $u_1, u_2, u_3, \ldots : u_{n+1}=\frac{1}{u_n}, u_1=\frac{2}{3}$	
(i) Way 1	$\left\{\sum_{r=1}^{16}(3+5r+2^r)=\right\}\sum_{r=1}^{16}(3+5r)+\sum_{r=1}^{16}(2^r)$	M1
	$=\frac{16}{2}(2(8)+15(5)) + \frac{2(2^{16}-1)}{2-1}$	M1 M1
	$=728+131070 = 131\,798$ *	A1*
		(4)
(i) Way 2	$\left\{\sum_{r=1}^{16}(3+5r+2^r)=\right\}\sum_{r=1}^{16}3+\sum_{r=1}^{16}(5r)+\sum_{r=1}^{16}(2^r)$	M1
	$=(3\times16)+\frac{16}{2}(2(5)+15(5)) + \frac{2(2^{16}-1)}{2-1}$	M1 M1
	$=48+680+131070 = 131\,798$ *	A1*
		(4)
(i) Way 3	Sum $= 10+17+26+39+60+97+166+299+560+1077+2106$ $+4159+8260+16457+32846+65619=131\,798$ *	M1 M1 M1 A1*
		(4)
(ii)	$\left\{u_1=\frac{2}{3}\right\}, u_2=\frac{3}{2}, u_3=\frac{2}{3}, \ldots$ ***(can be implied by later working)***	M1
	$\left\{\sum_{r=1}^{100}u_r=\right\}\ 50\left(\frac{2}{3}\right)+50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3}+\frac{3}{2}\right)$	M1
	$=\frac{325}{3}\ \left(\text{or } 108\frac{1}{3} \text{ or } 108.\dot{3} \text{ or } \frac{1300}{12} \text{ or } \frac{650}{6}\right)$	A1
		(3)

7(a)	$\sqrt{4-9x}=2(1\pm\ldots)^{\frac{1}{2}}$	B1
	$\left(1-\text{"}\frac{9x}{4}\text{"}\right)^{\frac{1}{2}}=\ldots+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\left(\text{"}-\frac{9x}{4}\text{"}\right)^2}{2!}$ or $\ldots+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\left(\text{"}-\frac{9x}{4}\text{"}\right)^3}{3!}$	M1
	$1+\frac{1}{2}\times\left(-\frac{9x}{4}\right)+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\left(-\frac{9x}{4}\right)^2}{2!}+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\left(-\frac{9x}{4}\right)^3}{3!}$	A1
	$\sqrt{4-9x}=2-\frac{9x}{4}-\frac{81x^2}{64}-\frac{729x^3}{512}$	A1
		(4)
(b)	States that the approximation will be an **<u>overestimate</u>** since all terms (after the first one) in the expansion are negative (since $x>0$)	B1
		(1)

4 (a)	$\frac{1}{\sqrt{4-x}}=(4-x)^{-\frac{1}{2}}=4^{-\frac{1}{2}}\times(1\pm\ldots\ldots$	M1
	Uses a "correct" binomial expansion for their $(1+ax)^n=1+nax+\frac{n(n-1)}{2}a^2x^2+$	M1
	$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)^2$	A1
	$\frac{1}{\sqrt{4-x}}=\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$	A1
		(4)
(b) (i)	States $x=-14$ and gives a valid reason. Eg explains that the expansion is not valid for $\lvert x\rvert>4$	B1
		(1)
(b)(ii)	States $x=-\frac{1}{2}$ and gives a valid reason. Eg. explains that it is closest to zero	B1
		(1)

11 (a)	$\sqrt{\frac{1+4x}{1-x}}=(1+4x)^{0.5}\times(1-x)^{-0.5}$	B1
	$(1+4x)^{0.5}=1+0.5\times(4x)+\frac{0.5\times-0.5}{2}\times(4x)^2$ $(1-x)^{-0.5}=1+(-0.5)(-x)+\frac{(-0.5)\times(-1.5)}{2}(-x)^2$ $(1+4x)^{0.5}=1+2x-2x^2$ **and** $(1-x)^{-0.5}=1+0.5x+0.375x^2$ oe	M1 M1 A1
	$(1+4x)^{0.5}\times(1-x)^{-0.5}=(1+2x-2x^2\ldots\ldots)\times\left(1+\frac{1}{2}x+\frac{3}{8}x^2\ldots\right)$ $=1+\frac{1}{2}x+\frac{3}{8}x^2+2x+x^2-2x^2+\ldots$ $=A+Bx+Cx^2$	dM1
	$=1+\frac{5}{2}x-\frac{5}{8}x^2\ldots\ldots$ *	A1*
		(6)
(b)	Expression is valid $\lvert x\rvert<\frac{1}{4}$ Should not use $x=\frac{1}{2}$ as $\frac{1}{2}>\frac{1}{4}$	B1
		(1)
(c)	Substitutes $x=\frac{1}{11}$ into $\sqrt{\frac{1+4x}{1-x}}\approx1+\frac{5}{2}x-\frac{5}{8}x^2$	M1
	$\sqrt{\frac{3}{2}}=\frac{1183}{968}$	A1
	(so $\sqrt{6}$ is) $\frac{1183}{484}$ or $\frac{2904}{1183}$	A1
		(3)

12. Vectors

9(a)	Attempts both $\lvert\overrightarrow{PQ}\rvert=\sqrt{2^2+3^2+(-4)^2}$ and $\lvert\overrightarrow{QR}\rvert=\sqrt{5^2+(-2)^2}$	M1	3.1a
	States that $\lvert\overrightarrow{PQ}\rvert=\lvert\overrightarrow{QR}\rvert=\sqrt{29}$ so $PQRS$ is a rhombus	A1	2.4
		(2)	
(b)	Attempts BOTH $\overrightarrow{PR}=\overrightarrow{PQ}+\overrightarrow{QR}=7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$ AND $\overrightarrow{QS}=-\overrightarrow{PQ}+\overrightarrow{PS}=3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$	M1	3.1a
	Correct $\overrightarrow{PR}=7\mathbf{i}+3\mathbf{j}-6\mathbf{k}$ and $\overrightarrow{QS}=3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$	A1	1.1b
	Correct method for area $PQRS$. E.g. $\frac{1}{2}\times\lvert\overrightarrow{PR}\rvert\times\lvert\overrightarrow{QS}\rvert$	dM1	2.1
	$=\sqrt{517}$	A1	1.1b
		(4)	
			(6 marks)

2	$OA=2\mathbf{i}+3\mathbf{j}-4\mathbf{k}$, $OB=4\mathbf{i}-2\mathbf{j}+3\mathbf{k}$, $OC=a\mathbf{i}+5\mathbf{j}-2\mathbf{k}$, $a<0$ $\overrightarrow{AB}=\overrightarrow{BD}$, $\lvert\overrightarrow{AB}\rvert=4$	
(a)	E.g. $\overrightarrow{OD}=\overrightarrow{OB}+\overrightarrow{BD}=\overrightarrow{OB}+\overrightarrow{AB}$ **or** $\overrightarrow{OD}=\overrightarrow{OB}+\overrightarrow{BD}=\overrightarrow{OB}+\overrightarrow{AB}=\overrightarrow{OB}+\overrightarrow{OB}-\overrightarrow{OA}=2\overrightarrow{OB}-\overrightarrow{OA}$ **or** $\overrightarrow{OD}=\overrightarrow{OB}+\overrightarrow{BD}=\overrightarrow{OB}+\overrightarrow{AB}=\overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{AB}=\overrightarrow{OA}+2\overrightarrow{AB}$	
	$=\begin{pmatrix}4\\-2\\3\end{pmatrix}+\begin{pmatrix}4\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\-4\end{pmatrix}\left\{=\begin{pmatrix}4\\-2\\3\end{pmatrix}+\begin{pmatrix}2\\-5\\7\end{pmatrix}\right\}$ **or** $=\begin{pmatrix}2\\3\\-4\end{pmatrix}+2\left(\begin{pmatrix}4\\-2\\3\end{pmatrix}-\begin{pmatrix}2\\3\\-4\end{pmatrix}\right)\left\{=\begin{pmatrix}2\\3\\-4\end{pmatrix}+2\begin{pmatrix}2\\-5\\7\end{pmatrix}\right\}$	M1
	$=\begin{pmatrix}6\\-7\\10\end{pmatrix}$ or $6\mathbf{i}-7\mathbf{j}+10\mathbf{k}$	A1
		(2)
(b)	$(a-2)^2+(5-3)^2+(-2--4)^2$	M1
	$\{\lvert\overrightarrow{AC}\rvert=4\Rightarrow\}\ (a-2)^2+(5-3)^2+(-2--4)^2=(4)^2$ $\Rightarrow(a-2)^2=8\Rightarrow a=\ldots$ **or** $\Rightarrow a^2-4a-4=0\Rightarrow a=\ldots$	dM1
	(as $a<0\Rightarrow$) $a=2-2\sqrt{2}$ (or $a=2-\sqrt{8}$)	A1
		(3)

13(a)	Attempts two of the relevant vectors $\pm\overrightarrow{AB}=\pm(-4\mathbf{i}+7\mathbf{j}+\mathbf{k})$ $\pm\overrightarrow{AC}=\pm(-20\mathbf{i}+(p+3)\mathbf{j}+5\mathbf{k})$ $\pm\overrightarrow{BC}=\pm(-16\mathbf{i}+(p-4)\mathbf{j}+4\mathbf{k})$	M1
	Uses two of the three vectors in such a way as to find the value of p. E.g. $p+3=5\times7$	dM1
	$p=32$	A1
		(3)
	(a) Alternative:	
	$r_{AB}=4\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\lambda(-4\mathbf{i}+7\mathbf{j}+\mathbf{k})$	M1
	$4\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\lambda(-4\mathbf{i}+7\mathbf{j}+\mathbf{k})=-16\mathbf{i}+p\mathbf{j}+10\mathbf{k}\Rightarrow\lambda=5$ $4\mathbf{i}-3\mathbf{j}+5\mathbf{k}+\lambda(-4\mathbf{i}+7\mathbf{j}+\mathbf{k})=-16\mathbf{i}+p\mathbf{j}+10\mathbf{k}\Rightarrow p=35-3$	dM1
	$p=32$	A1
(b)	Deduces that $\overrightarrow{OD}=\lambda\overrightarrow{OB}=4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts $\overrightarrow{CD}=16\mathbf{i}+(4\lambda-\text{"}32\text{"})\mathbf{j}+(6\lambda-10)\mathbf{k}$	M1
	Correct attempt at λ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$ $\overrightarrow{CD}=16\mathbf{i}+(4\lambda-\text{"}32\text{"})\mathbf{j}+(6\lambda-10)\mathbf{k}$ $\overrightarrow{OA}=4\mathbf{i}-3\mathbf{j}+5\mathbf{k}$ $4\lambda-32=-12\Rightarrow\lambda=\ldots$ OR $6\lambda-10=20\Rightarrow\lambda=\ldots$	dM1
	$\lvert\overrightarrow{OD}\rvert=5\times\sqrt{4^2+6^2}=10\sqrt{13}$	A1
		(3)
	(b) Alternative:	
	Deduces that $\overrightarrow{OD}=\lambda\overrightarrow{OB}=4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts $\overrightarrow{OD}=\overrightarrow{OC}+\mu\overrightarrow{OA}=-16\mathbf{i}+32\mathbf{j}+10\mathbf{k}+\mu(4\mathbf{i}-3\mathbf{j}+5\mathbf{k})$	M1
	Correct attempt at λ **or** μ using the fact that $\lambda\overrightarrow{OB}=\overrightarrow{OC}+\mu\overrightarrow{OA}$ E.g. $-16+4\mu=0\Rightarrow\mu=4$	dM1
	$\lvert\overrightarrow{OD}\rvert=5\times\sqrt{4^2+6^2}=10\sqrt{13}$	A1
		(3)

10. Numerical methods

5(a)	States or uses $h=1.5$	B1
	Full attempt at the trapezium rule $=\frac{\cdots}{2}\{1.63+2.63+2\times(2+2.26+2.46)\}$	M1
	$=$ awrt 13.3 or $\frac{531}{40}$	A1
		(3)
(b)(i)	$\int_3^9 \log_3(2x)^{10}\,dx=10\times\text{"13.3"}=\text{awrt}\,133$ or e.g. $\frac{531}{4}$	B1ft
(ii)	$\int_3^9 \log_3 18x\,dx=\int_3^9 \log_3(9\times 2x)\,dx=\int_3^9 2+\log_3 2x\,dx$ $=[2x]_3^9+\int_3^9 \log_3 2x\,dx=18-6+\int_3^9 \log_3 2x\,dx=\ldots$	M1
	Awrt 25.3 or $\frac{1011}{40}$	A1ft
		(3)

6(a)	$(\mathrm{f}'(x)=)4\cos\left(\frac{1}{2}x\right)-3$	M1 A1
	Sets $\mathrm{f}'(x)=4\cos\left(\frac{1}{2}x\right)-3=0\Rightarrow x=$	dM1
	$x=14.0$ Cao	A1
		(4)
(b)	Explains that $\mathrm{f}(4)>0$, $\mathrm{f}(5)<0$ **and** the function is continuous	B1
		(1)
(c)	Attempts $x_1=5-\frac{8\sin 2.5-15+9}{\text{"}4\cos 2.5-3\text{"}}$ (NB $\mathrm{f}(5)=-1.212\ldots$ and $\mathrm{f}'(5)=-6.204\ldots$)	M1
	$x_1=$ awrt 4.80	A1
		(2)

4 (a)	Attempts $\mathrm{f}(3)=$ and $\mathrm{f}(4)=$ where $\mathrm{f}(x)=\pm(2\ln(8-x)-x)$	M1
	$\mathrm{f}(3)=(2\ln(5)-x)=(+)0.22$ and $\mathrm{f}(4)=(2\ln(4)-4)=-1.23$ Change of sign and function continuous in interval $[3,4]\Rightarrow$ Root *	A1*
		(2)
(b)	For annotating the graph by drawing a cobweb diagram starting at $x_1=4$ It should have at least two spirals	M1
	Deduces that the iteration formula **can be used** to find an approximation for α because **the cobweb spirals inwards** for the cobweb diagram	A1
		(2)

5	The equation $2x^3+x^2-1=0$ has exactly one real root	
(a)	$\{\mathrm{f}(x)=2x^3+x^2-1\Rightarrow\}\ \mathrm{f}'(x)=6x^2+2x$	
	$\left\{x_{n+1}=x_n-\frac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}\Rightarrow\right\}\ \{x_{n+1}\}=x_n-\frac{2x_n^3+x_n^2-1}{6x_n^2+2x_n}$	
	$=\frac{x_n(6x_n^2+2x_n)-(2x_n^3+x_n^2-1)}{6x_n^2+2x_n}\Rightarrow x_{n+1}=\frac{4x_n^3+x_n^2+1}{6x_n^2+2x_n}$ *	
(b)	$\{x_1=1\Rightarrow\}\ x_2=\frac{4(1)^3+(1)^2+1}{6(1)^2+2(1)}$ or $x_2=1-\frac{2(1)^3+(1)^2-1}{6(1)^2+2(1)}$	
	$\Rightarrow x_2=\frac{3}{4},\ x_3=\frac{2}{3}$	
(c)	Accept any reasons why the Newton-Raphson **method** cannot be used with $x_1=0$ which refer or ***allude*** to either the stationary point or the tangent. E.g. • There is a stationary point at $x=0$ • Tangent to the curve (or $y=2x^3+x^2-1$) would not meet the x-axis • Tangent to the curve (or $y=2x^3+x^2-1$) is horizontal	