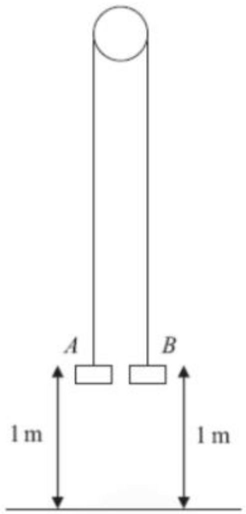
AS Mechanics – 11/03/24 - Questions

Question 3



Two particles A and B have mass 0.4 kg and 0.3 kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed above a horizontal floor. Both particles are held, with the string taut, at a height of 1m above the floor, as shown in the diagram above. The particles are released from rest and in the subsequent motion B does not reach the pulley.

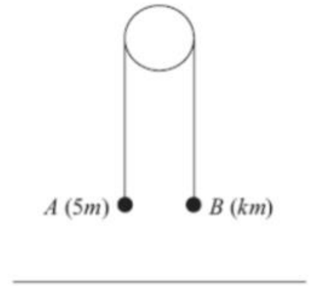
(a) Find the tension in the string immediately after the particles are released.

(b) Find the acceleration of A immediately after the particles are released.

When the particles have been moving for 0.5 s, the string breaks.

(c) Find the further time that elapses until B hits the floor.

Question 16



Two particles A and B have masses $5m$ and km respectively, where $k < 5$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane, as shown in Figure 4. The system is released from rest. After release, A descends with acceleration $\frac{1}{4}g$.

(a) Show that the tension in the string as A descends is $\frac{15}{4}mg$.

(3)

(b) Find the value of k.

(3)

(c) State how you have used the information that the pulley is smooth.

(1)

After descending for 1.2 s, the particle A reaches the plane. It is immediately brought to rest by the impact with the plane. The initial distance between B and the pulley is such that, in the subsequent motion, B does not reach the pulley.

(d) Find the greatest height reached by B above the plane.

(7)
(Total 14 marks)

Question 3

A car which has run out of petrol is being towed by a breakdown truck along a straight horizontal road. The truck has mass 1200 kg and the car has mass 800 kg. The truck is connected to the car by a horizontal rope which is modelled as light and inextensible. The truck's engine provides a constant driving force of 2400 N. The resistances to motion of the truck and the car are modelled as constant and of magnitude 600 N and 400 N respectively. Find

(a) the acceleration of the truck and the car,

(b) the tension in the rope.

When the truck and car are moving at 20 m s^{-1}, the rope breaks. The engine of the truck provides the same driving force as before. The magnitude of the resistance to the motion of the truck remains 600 N.

(c) Show that the truck reaches a speed of 28 m s^{-1} approximately 6 s earlier than it would have done if the rope had not broken.

Question 1

Two particles A and B, of mass m and $2m$ respectively, are attached to the ends of a light inextensible string. The particle A lies on a rough horizontal table. The string passes over a small smooth pulley P fixed on the edge of the table. The particle B hangs freely below the pulley, as shown in the diagram above. The coefficient of friction between A and the table is μ. The particles are released from rest with the string taut. Immediately after release, the magnitude of the acceleration of A and B is $\frac{4}{9}g$. By writing down separate equations of motion for A and B,

(a) find the tension in the string immediately after the particles begin to move,

(b) show that $\mu = \frac{2}{3}$.

When B has fallen a distance h, it hits the ground and does not rebound. Particle A is then a distance $\frac{1}{3}h$ from P.

(c) Find the speed of A as it reaches P.

(d) State how you have used the information that the string is light.

Question 6

A particle P moves on the x-axis. At time t seconds the velocity of P is v m s^{-1} in the direction of x increasing, where v is given by

$$v = \begin{cases} 8t - \frac{3}{2}t^2, & 0 \le t \le 4, \\ 16 - 2t, & t > 4. \end{cases}$$

When $t = 0$, P is at the origin O.

Find

(a) the greatest speed of P in the interval $0 \le t \le 4$,

(b) the distance of P from O when $t = 4$,

(c) the time at which P is instantaneously at rest for $t > 4$,

(d) the total distance travelled by P in the first 10 s of its motion.

Question 7

A particle P moves in a horizontal plane. At time t seconds, the position vector of P is $\mathbf{r}$ metres relative to a fixed origin O, and $\mathbf{r}$ is given by

$$\mathbf{r} = (18t - 4t^3)\mathbf{i} + ct^2\mathbf{j},$$

where c is a positive constant. When $t = 1.5$, the speed of P is 15 m s^{-1}. Find

(a) the value of c,

(b) the acceleration of P when $t = 1.5$.

Question 8

A cricket ball of mass 0.5 kg is struck by a bat. Immediately before being struck, the velocity of the ball is $(-30\mathbf{i})$ m s^{-1}. Immediately after being struck, the velocity of the ball is $(16\mathbf{i} + 20\mathbf{j})$ m s^{-1}.

(a) Find the magnitude of the impulse exerted on the ball by the bat.

In the subsequent motion, the position vector of the ball is $\mathbf{r}$ metres at time t seconds. In a model of the situation, it is assumed that $\mathbf{r} = [16t\mathbf{i} + (20t - 5t^2)\mathbf{j}]$. Using this model,

(b) find the speed of the ball when $t = 3$.

8.

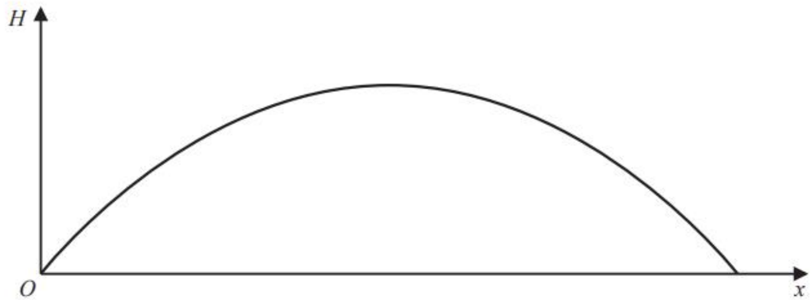


Figure 1

Figure 1 is a graph showing the trajectory of a rugby ball.

The height of the ball above the ground, H metres, has been plotted against the horizontal distance, x metres, measured from the point where the ball was kicked.

The ball travels in a vertical plane.

The ball reaches a maximum height of 12 metres and hits the ground at a point 40 metres from where it was kicked.

(a) Find a quadratic equation linking H with x that models this situation.

(3)

The ball passes over the horizontal bar of a set of rugby posts that is perpendicular to the path of the ball. The bar is 3 metres above the ground.

(b) Use your equation to find the greatest horizontal distance of the bar from O.

(3)

(c) Give one limitation of the model.

(1)

9.

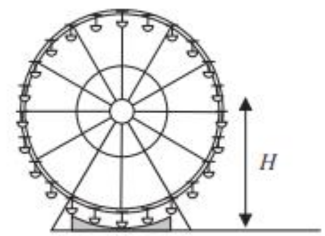


Figure 4

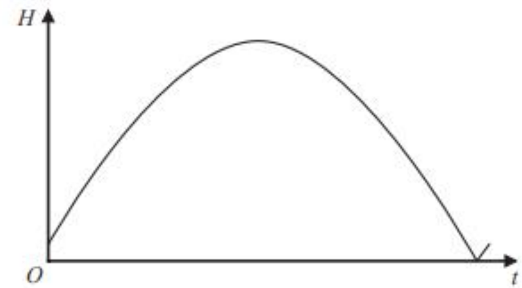


Figure 5

Figure 4 shows a sketch of a Ferris wheel.

The height above the ground, H m, of a passenger on the Ferris wheel, t seconds after the wheel starts turning, is modelled by the equation

$$H = |A\sin(bt + \alpha)^\circ|$$

where A, b and α are constants.

Figure 5 shows a sketch of the graph of H against t, for one revolution of the wheel.

Given that

- the maximum height of the passenger above the ground is 50 m
- the passenger is 1 m above the ground when the wheel starts turning
- the wheel takes 720 seconds to complete one revolution

(a) find a complete equation for the model, giving the exact value of A, the exact value of b and the value of α to 3 significant figures.

(4)

(b) Explain why an equation of the form

$$H = |A\sin(bt + \alpha)^\circ| + d$$

where d is a positive constant, would be a more appropriate model.

(1)

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Question 3

$m_A > m_B \Rightarrow$ A goes down and B up.

a)

on A: $m_A g - T = m_A a$ $\quad 0.4g - T = 0.4a$ (1) $\times 30$

on B: $T - m_B g = m_B a$ $\quad T - 0.3g = 0.3a$ (2) $\times 40$

$$\begin{cases} 12g - 30T = 12a \\ 40T - 12g = 12a \end{cases} \Rightarrow \quad 12g - 30T = 40T - 12g$$

$70T = 24g$

$T = \frac{12}{35} g$ N $\quad (g = 9.8\ m/s^2)$

b) (1) $0.4g - \frac{12}{35} g = 0.4a$ or: (1)+(2): $0.4g - 0.3g = 0.7a$

$0.1g = 0.7a \Rightarrow a = \frac{g}{7} = 1.4\ m/s^2$

c) After 0.5 s accelerating at $1.4\ m/s^2$, the speed of A and B is 0.7 m/s.

B has gone up by: $s = ut + \frac{1}{2}at^2$ $\quad (u = 0)$

$= \frac{1}{2} \times 1.4 \times \left(\frac{1}{2}\right)^2$

$= 0.175$ m

So B is now $1 + 0.175 = 1.175$ m above the floor.

It now falls with an acceleration of $a = g = 9.8\ m/s^2$:

$s = ut + \frac{1}{2}at^2$ $\quad$ ($u = -0.7\ m/s$ if ↓ is positive direction,

$1.175 = -0.7t + 4.9t^2$

$4.9t^2 - 0.7t - 1.175 = 0 \Rightarrow t = \frac{0.7 \pm \sqrt{0.7^2 + 4 \times 4.9 \times 1.175}}{9.8}$

$t > 0 \Rightarrow t = 0.5663\ldots$ s

Question 16

$k < 5 \Rightarrow$ A is heavier than B $\Rightarrow$ A goes down and B up.

a) forces on A $\quad 5mg - T = 5ma$

" on B $\quad T - kmg = kma$

with $a = \frac{1}{4}g$, we get:

$5mg - T = \frac{5mg}{4}$ (1) $\Rightarrow T = 5mg - \frac{5mg}{4}$

$T - kmg = \frac{kmg}{4}$ (2) $\quad \boxed{T = \frac{15}{4} mg}$

b) Sub $T = \frac{15}{4} mg$ in (2) $\Rightarrow \frac{15}{4} mg - kmg = \frac{kmg}{4}$

$\frac{15}{4} - k = \frac{k}{4}$

$15 - 4k = k$

$\boxed{k = 3}$

c) Pulley is smooth => no friction, no energy loss and so the tension in the string is the same in the two parts of the string.

d) In 1.2 s at $a = \frac{1}{4}g$, the displacement is:

$s_1 = \frac{1}{2}at^2$ $\quad (u = 0)$

$= \frac{1}{2} \times \frac{1}{4}g \times 1.2^2$

$= 0.18g$ m = distance of A and B above the ground when they are released.

Speed of B after 1.2s:

$v = at = \frac{1}{4}g \times 1.2 = 0.3g\ m/s$ (going up)

Then B falls with $a = g$. Greatest height reached when $v = 0 \Rightarrow$

$u + at = -0.3g + gt = 0$ (down is positive direction)

$\Rightarrow t = 0.3$ s $\Rightarrow s_2 = ut + \frac{1}{2}at^2 = -0.3g \times 0.3 + \frac{1}{2}g\, 0.3^2$

$= \frac{0.09}{2} g$ m

$\Rightarrow S = s_1 + s_2 = 3.969 \simeq 4.0$ m

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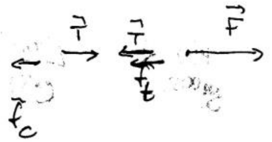
Question 3



a) Forces on car: $T - f_c = m_c a$ (1) $\Rightarrow$ $T = m_c a + f_c$

" " truck: $F - T - f_t = m_t a$ (2)

(2): $F - (m_c a + f_c) - f_t = m_t a$

$2400 - (800a + 400) - 600 = 1200a$

$2000a = 1400$

$a = 0.7 \ m/s^2$

b) $T = 800a + 400 = 960 \ N$

c) Forces on truck: $F - f_t = m_t a$

$2400 - 600 = 1200 a$

$\Rightarrow a = 1.5 \ m/s^2$

$\Delta v = 28 - 20 = 8 \ m/s = at$

$\Rightarrow t = \frac{\Delta v}{a}$

if $a = 1.5 \ m/s^2 \Rightarrow t = \frac{8}{1.5} = 5\frac{1}{3}$ s.

if $a = 0.7 \ m/s^2 \Rightarrow t = \frac{8}{0.7} = \frac{80}{7} = 11\frac{3}{7}$ s

Difference : $11\frac{3}{7} - 5\frac{1}{3} \simeq 11 - 5 \simeq 6$ s.

Question 1

a) Horizontal forces on A:

$T - f_A = m_A a = m_A \times \frac{4}{9} g$

$T - \mu mg = \frac{4}{9} mg$ (1)

Forces on B: $2mg - T = 2m \times \frac{4}{9} g$

$\Rightarrow T = \left(2 - \frac{8}{9}\right) mg = \frac{10}{9} mg$

b) Sub in (1): $\frac{10}{9} mg - \mu mg = \frac{4}{9} mg$

$\mu = \frac{10}{9} - \frac{4}{9} = \frac{6}{9} = \frac{2}{3}$

c) When B hits the floor, its speed (it's also the speed of A at this moment) is: $v^2 = 2 \times ah = 2 \times \frac{4g}{9} \times h$

Once B is on the floor, A moves with deceleration due to the friction force $f_A = \frac{2}{3} mg \Rightarrow a = \frac{2}{3} g$

Speed of A at P: $v_A^2 = \frac{8gh}{9} - 2 \times \frac{2g}{3} \times \frac{h}{3} = \frac{4gh}{9}$

$\Rightarrow v_A = \frac{2}{3}\sqrt{gh}$

d) String is light so same tension on A and B.

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Question 6

a) Greatest speed when $\frac{dv}{dt} = a = 0$

$a = 8 - 3t$ for $0 \leq t \leq 4$

$= 0$ at $t = \frac{8}{3}$ s $\Rightarrow v = 8 \times \frac{8}{3} - \frac{3}{2} \times \left(\frac{8}{3}\right)^2 = \frac{32}{3}$ m/s

b) for $0 \leq t \leq 4$, $s = 4t^2 - \frac{1}{2} t^3$

at $t = 4$, $s = 4 \times 4^2 - \frac{1}{2} \times 4^3 = 32$ m

c) Instantaneously at rest means $v = 0$ for a particular t.

$16 - 2t = 0 \Rightarrow t = 8$ s.

d) First 4 s: $d_1 = 32$ m (Q6)

Then $v \geq 0$ between 4 and 8 s. Area under the line

$v = 16 - 2t$ is : $\frac{1}{2} \times 8 \times (8-4) = 16$ m.

for $8 \leq t \leq 10$, the area under the line is : $\frac{1}{2} \times 4 \times (10 - 8) = 4$

$\Rightarrow$ Total distance $= 32 + 16 + 4 = 52$ m.

Question 7

a) From $\vec{r} = (18t - 4t^3)\vec{i} + ct^2\vec{j}$, we find that

$\vec{v} = \frac{d\vec{r}}{dt} = (18 - 12t^2)\vec{i} + 2ct\,\vec{j}$.

At $t = \frac{3}{2}$ s, we find that $\vec{v} = \left(18 - 12 \times \left(\frac{3}{2}\right)^2\right)\vec{i} + 2c \times \frac{3}{2}\vec{j}$

$= -9\vec{i} + 3c\vec{j}$.

is $|\vec{v}| = 15$ m/s $= \sqrt{81 + 9c^2}$

$81 + 9c^2 = 15^2 \Rightarrow 9c^2 = 144 \rightarrow c = 4$

b) $\vec{a} = \frac{d\vec{v}}{dt} = -24t\,\vec{i} + 8\vec{j}$

at $t = \frac{3}{2}$ s, $\vec{a} = -24 \times \frac{3}{2}\vec{i} + 8\vec{j} = -36\vec{i} + 8\vec{j}$

Question 8

a) Impulse $= \Delta(m\vec{v}) = m(\vec{v}_f - \vec{v}_i)$

$= 0.5\,(16\vec{i} + 20\vec{j} - (-30\vec{i}))$

$= 0.5\,(46\vec{i} + 20\vec{j})$

$= 23\vec{i} + 10\vec{j}$

$\|\Delta(m\vec{v})\| = \sqrt{23^2 + 10^2} \simeq 25.1$ N.s

b) If $\vec{r} = 16t\,\vec{i} + (20t - 5t^2)\vec{j}$, then

$\vec{v} = \frac{d\vec{r}}{dt} = 16\vec{i} + (20 - 10t)\vec{j}$

At $t = 3$ s, $\vec{v} = 16\vec{i} - 10\vec{j}$

$\Rightarrow v = \|\vec{v}\| = \sqrt{16^2 + 10^2} \simeq 18.9$ m/s